First International Conference on Application
of Mathematics in Technical and Natural Sciences
22–27 June 2009, Sozopol, Bulgaria

BOOK OF ABSTRACTS

Sofia, Lafayette • 2009

Modeling Hemodynamics of Coronary Blood Flow during Atherosclerosis

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The function of the coronary network is to supply blood to the heart however in cases of coronary Artery disease (CAD), transport phenomena and geometry has much influence on blood flow dynamics and the overall performance of the heart. In this paper, blood is modeled as a non-Newtonian fluid and the flow behavior is described using Eyring-Powell model. The momentum equation for the flow is non-dimensionalized and the non-linear dimensionless equation is then solved numerically by shooting method. Variations of different flow parameters are conducted to give insight into the hemodynamics of blood. Prominent variations in the velocity profile are shown and discussed.

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Approximations of Sturm-Liouville Eigenvalues Using Sinc-Galerkin and Differential Transform Methods

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In this paper, we present a comparative study between Sinc-Galerkin method and differential transform method to solve Sturm-Liouville eigenvalue problem. As an application, a comparison between the two methods for various celebrated Sturm-Liouville problems are analyzed for their eigenvalues and solutions. The study outlines the significant features of the two methods. The results show that these methods are very efficient, convenient and can be applied to a large class of problems. The comparison of the methods shows that although the numerical results of these methods are the same, differential transform method is much easier, more convenient and efficient than the Sinc-Galerkin method.

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Approximate Solutions to Nonlinear Partial Integro-Differential Equations with Applications in Heat Flow

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In this paper, two different methods based on variational iteration method (VIM) and on differential transform method (DTM) are developed to approximate solutions of some partial integro-differential equations with applications in heat flow. Approximate solutions are obtained for some important physical problems concerned with heat flow in materials with memory. The methods are capable of greatly reducing the size of computational domain. Some numerical examples are presented to show the performances and accuracy of the proposed methods.

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3D Scene Restoration Using One Active PTZ Camera

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The active PTZ (Pan Tilt Zoom) camera is a key element of an intelligent surveillance system. The opportunity to control camera parameters significantly increases the abilities of these cameras as information sources. It is common regarded that a camera gives 2D presentation of 3D scene. The depth of the scene is irrevocably lost and only some image features may indirectly reveal the position of the objects in third dimension. The active camera can partially overcome this loss of information. The suitable control of camera parameters may be used for estimation of the depth of the observed objects. There are several approaches to reconstruct 3D scene. The authors of the paper realized the most practical one using limited number of camera frames and investigate its characteristics. The presented results include also recommendations how to reach better accuracy in 3D scene restoration.

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Hybrid Finite Element and Finite Volume Methods for Free-Surface Flows

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We have successfully extended our implicit hybrid finite element/volume solver to flows involving two immiscible fluids. The solver is based on the segregated pressure correction or projection method on staggered unstructured hybrid meshes. An intermediate velocity field is first obtained by solving the momentum equations with the matrix-free implicit cell-centered finite volume method. The pressure Poisson equation is solved by the node-based Galerkin finite element method for an auxiliary variable. The auxiliary variable is used to update the velocity field and the pressure field. The pressure field is carefully updated by taking into account the velocity divergence field. This updating strategy can be rigorously proven to be able to eliminate the unphysical pressure boundary layer and is crucial for the correct temporal convergence rate. Our current staggered-mesh scheme is distinct from other conventional ones in that we store the velocity components at cell centers and the auxiliary variable at vertices. The fluid-interface is captured by solving an advection equation for the volume fraction of one of the fluids. The same matrix-free finite volume method as the one used for momentum equations is used to solve the advection equation. We will focus on the interface sharpening strategy to minimize the smearing of the interface over time. We have developed and implemented a global mass conservation algorithm which enforces the conservation of the mass for each fluid. 3D application problem includes air-water flow passed a composite high speed vehicle with lifting hydrofoils at high speeds.

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Finite Element Solution of 2D and 3D Elliptic Problems with Intersected Interfaces

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In this article we present second-order finite element (FE) symmetric approximations for 2D and 3D elliptic problems with intersecting interfaces. The discretizations are derived, using two type (2D case) and six type (3D case) linear FE with averaging of the obtained FEM approximations at each grid node. The convergence and accuracy are discussed theoretically and experimentally. Numerical tests show the feasibility of the schemes.

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Continuous Age-Structured Model of Bovine Tuberculosis in African Buffalo

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The paper deals with a model of the spread of bovine tuberculosis in the buffalo population in the Kruger National Park in South Africa. The model uses continuous age structure and it is formulated in terms of partial differential equations using eight epidemiological classes (compartments). More precisely, the age density for each class at time t satisfies a one way wave equation, where the age is the space variable. The continuous age model discussed here is derived from a 2006 age groups model by P. C. Cross and W. M. Getz. The model of Cross and Getz is formulated as a system of difference equations using 18 age groups and a total of 144 compartments. These are replaced in this work by 8 partial differential equations, making it more convenient for both theoretical investigations and numerical simulations.

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SALUTE Grid Application Using and Vacuum: Message-Oriented Middleware

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Stochastic ALgorithms for Ultra-fast Transport in sEmiconductors (SALUTE) is a grid application developed for solving various computationally intensive problems which describe ultrafast carrier transport in semiconductors. SALUTE studies memory and quantum effects during the relaxation process due to electronphonon interaction in one-band semiconductors or quantum wires. Formally, SALUTE integrates a set of novel Monte Carlo, quasi-Monte Carlo and hybrid algorithms for solving various computationally intensive problems which describe the femtosecond relaxation process of optically excited carriers in one-band semiconductors or quantum wires. In this paper we present application – specific job submission and monitoring tool – Job Track Service (JTS) which is developed using Message-Oriented middleware to implement robust, versatile job submission and tracing mechanism, which can be tailored to application specific failover and quality of service requirements. Experience from using the JTS for submission of SALUTE jobs is presented.

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Amplitude-Phase Formulation for a Discrete Two Species Lotka-Volterra System

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There are many papers in which different types of difference schemes for the two species Lotka-Volterra (LV) predator-prey system $\dot{x} = x - xy$, $\dot{y} = -y + xy$ are considered (see, for example, [1]–[5]). The dynamics of the differential LV model is well-known. The following are the main properties of the system:

(1) The solutions are positive and periodic if the initial conditions are positive.

(2) The average of $x(t)$ and $y(t)$, evaluated over a period T, coincides with the coordinates of the fixed point (1;1), $\bar{x} = 1$, $\bar{y} = 1$.

The systems of two difference equations, obtained in [1]–[5], produce periodic solutions about the fixed point (1, 1). However, there is a dependence on the fixed step-size h of a period T and the conditions $\bar{x} = 1$, $\bar{y} = 1$ are not satisfied. The reason for this is the following. If $h = \text{const}$ the most part of iteration points of LV system belongs to the region where x and y vary slowly over time. Therefore a fixed constant angle step LV model will be better than the fixed constant time step model.

The aim of this work is to show how the concept of nonlinear oscillations can be used to characterize periodic orbits of LV system. In line with previous studies by Waldvogel [6], a nonlinear xy LV model, by means of some transformations, is written in terms of amplitude A and phase ϕ, where A is a constant of motion and ϕ is amplitude dependent:

$$\frac{dA}{d\varphi} = 0, \qquad \frac{dt}{d\varphi} = F(A, \phi). \tag{1}$$

The equations (1) generate the following discrete in ϕ LV model

$$A_i = A_0, \qquad t_i = \int_{\phi_0}^{\phi_i} F(A_0, \phi) d\phi,$$

where $\phi_i = \phi_0 + \frac{\pi i}{2N}$, $i = 1, 2, \ldots$. This method preserves the oscillation feature of the discrete predator-prey system and its period, as well as sufficiently uniform distribution of x and y values.

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Modeling the Effects of Mobile Phone on the Development of Skin Cancer

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The New information and telecommunication technology comes with innovat-ions as well as far negative consequences. The mobile phones we used for our daily business is having a negative effect on our health depending on the way we use the device. We modeled the contact effects of mobile phones on the body (skin) using reaction diffusion equations. The model predicts early development of cancer of the skin on weakly immune individuals.

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Mathematical Tools for Energy Production, Transportation and Storage

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We give a general overview of the type of mathematical tools which are useful in the energy sector. A general concern about these tools is that they should be "robust", in the sense that they should take into account various uncertainties for the future, imprecise or missing data for the past. We show how probabilistic methods can be used, without making arbitrary assumptions.

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Natural Gas Consumption, Importation and Storage

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Following some contracts we had with the French Government in 2006-2007, we show how mathematical tools may be used in order to answer questions such as : are our imports sufficient, are our stocks sufficient? Since consumption is linked with temperature, we show how probabilistic laws for temperature must be used, how they allow a probabilistic prediction of the consumption on any particular day of the year, and how this leads to an estimate upon the level of the stocks.

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Mathematics for Investments Planning in Electricity Transportation

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New nuclear power plants will be built, the electricity consumption will increase, and some equipments may become obsolete or dangerous. Therefore, there is a need to plan new investments. But the definition of this planning is very sensitive and complex, as it should include some estimate upon the cost of a breakdown (no electricity in some zone for some time). We show how to use probabilistic methods in order to introduce a contract of quality with the consumers.

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Super-resolution with the Help of Active Camera

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In a digital optical imaging system, image resolution is constrained by several factors, including focus plane array pitch and optics. Super-resolution approaches aim to overcome some of this limits by incorporating additional information of the object and/or combining several pictures of the same object, taken with sub-pixel displacements between each other.

As shown by some authors, the obtainable zoom factor using the second one has an upper limit, determined by the signal-to-noise ratio of the image taken. Moreover, as discussed later, lines at some pitch are indistinguishable, further lowering this limit. Here, an approach to overcome this using active camera and employing images of different relative scales is introduced. The choice of these scales is discussed. Then some experiments on synthetic and real data are presented and the obtained results are presented and discussed.

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Electro-Thermal Simulation and Analysis of The New Technology 3D Die-Stacked Integrated Circuits

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The new technology of three-dimensional integrated circuits (3D ICs) stacks active layers of transistors one above the other, separated by insulating oxide, and connected to each other by metal interconnect wires. This has the advantage of reducing significantly wire lengths, increasing speed, and provide lower power consumption. However, as we stack more transistors, the power density increases causing the temperatures to increase. In this paper, we simulate the electro-thermal heating of multiple layered MOSFETs. We use mathematical models based on drift-diffusion equations coupled with heat equation involving different heat sources. We show that, compared to a single active layer, the multiple-layered structure increases the maximum temperature significantly depending on active layer thickness and wiring thermal boundary conditions. Numerical results and a typical structure showing substantial thermal increase in MOSFETs away from the heat sink will be presented.

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Problems of Wave Diffraction by Quadrants Analysed by a Half-line Potential Approach

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We will consider problems of wave diffraction by quadrants with general boundary conditions. A method based on certain half-line potential operators will be introduced to study these problems. In particular, a class of boundary value problems for the Helmholtz equation where two boundary conditions are given on the half-lines bordering the first quadrant that contain oblique derivatives will be presented in detail. Moreover, the boundary pseudodifferential operators which characterize the corresponding problems for the Helmholtz equation are reduced by matricial coupling relations to certain compositions of Wiener-Hopf operators. This allows the explicit construction of the corresponding (lateral) inverses in Bessel potential spaces, eventually after normalization, and regularity results.

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Fredholm Analysis of a Wave Diffraction Problem with Higher Order Boundary Conditions

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We consider a problem of wave diffraction characterized with higher order boundary conditions. From the mathematical point of view, the problem is formulated as a boundary-transmission problem for the Helmholtz equation within a Bessel potential space framework. Operator theoretical methods are used to deal with this problem and, as a consequence, several convolution type operators are constructed and associated to the problem. A Fredholm characterization of these operators is obtained for certain smoothness space orders, and a consequent mathematical analysis of the initial problem is derived.

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A Regularization Technique for Numerical Solution of an Initial-Boundary Value Problem in Price Formation

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The paper, P.-L. Lions and J-M. Lasry, Ann. I. H. Poincare-AN 24 (2007) 311-323, is devoted to the effect of reconciling the classical Black-Sholes theory of option pricing and hedging with various phenomena observed in the markets such as the influence of trading and hedging on the dynamics of an asset. Here we shall discuss the numerical solution of initial boundary-value problems to the model equation of the theory

$$\frac{\partial f}{\partial z} - \frac{\sigma^2}{2}\frac{\partial^2 f}{\partial x^2} = \lambda(t)[\delta_{x=-a} \quad - \delta_{x=a}], \lambda(t) := -\frac{\sigma^2}{2}\frac{\partial f}{\partial x}(0,t).$$

The symbol δ denotes the Dirac delta at the indicated point, f is a price non-negative density, σ is constant volatility, and λ represents the number of transactions at time t.

The lack of regularity in the solution as result from Diract delta reduces the accuracy in the numerical computations. We shall implement a technique of local regularization introduced by A-K. Tornberg and B. Engquist for handling this equation. A priori bounds are obtained for the difference equations that imply stability and convergence of difference schemes for the problems under consideration. Numerical experiments are discussed.

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Numerical Investigation of the 2D Paradigm Boussinesq Equation

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One of the most important features of generalized wave equations containing nonlinearity and dispersion, is that they possess solutions of type of permanent waves which behave in many instances as particles. When the governing system is fully integrable, such waves are called solitons. In 1D a plethora of deep mathematical results have been obtained for solitons. The success was contingent upon the existence of an analytical solution of the respective nonlinear dispersive equation. Naturally, predominant part of the theoretical results were confined to the 1D case. It is of high importance to investigate the 2D case which, in most of the cases, can be done only numerically. Here we investigate numerically the 2D Boussinesq Paradigm equation using a Galerkin spectral method. The method employs a system of functions defined on $L^2(-\infty, \infty)$ with exponential rate of convergence. The method has been used in others works of the authors and has been proven to be a very good numerical tool for problems in infinite domains. To construct the numerical scheme we use the method of operator splitting. Even though the scheme is not fully implicit its margin of stability is wide enough and allows optimization of the rate of convergence.

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A Two-grid Fictitious Domain Method for Direct Simulation of Flows Involving Particles of a Very Small Size

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The full resolution of flows involving particles whose scale is hundreds or thousands of times smaller than the size of the flow domain is a challenging problem. A naive approach would require a tremendous number of degrees of freedom in order to bridge the gap between the two spatial scales involved. The approach used in the present study employs two grids whose grid size fits the two different scales involved, one of them (the micro-scale grid) being embedded into the other (the macro-scale grid). Then resolving first the larger scale on the macro-scale grid, we transfer the so obtained data to the boundary of the micro-scale grid and solve the smaller size problem. Since the particle is moving throughout the macro-scale domain, the micro-scale grid is fixed at the centroid of the moving particle and therefore moves with it. In this study we combine such an approach with a fictitious domain formulation of the problem (see [1], resulting in a very efficient algorithm that is also easy to implement in an existing CFD code. We validate the method against existing experimental data for a sedimenting sphere, as well as analytical results for motion of an inertialess ellipsoid in a shear flow. Finally, we apply the method to the flow of a high aspect ratio ellipsoid in a model of a human lung airway bifurcation.

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Explicit Computation of the Cauchy-Riesz-Dunford Operator Integral in the Finite-dimensional Case

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Let $B = B_N$ be a finite-dimensional Banach space of dimension N over one complex field, let T be a bounded linear operator on B and assume that the function $f = f(z)$, $z \in \mathbf{C}$, is analytic in a domain $\Omega \subset C$ containing the closure of an open set containing the spectrum $\sigma(T)$ of T. Suppose also that the boundary $\partial\Omega$ of Ω consists of a finite number of closed rectifiable Jordan curves oriented in the positive sense customary in the theory of complex variables. We prove the following identity

$$\begin{aligned} f(T) =& \frac{1}{2\pi i} \int\limits_{\partial\Omega} f(\lambda)(\lambda I - T)^{-1} d\lambda \\ =& \sum_{\mu=1}^{M} \left[\sum_{\nu=0}^{K_\mu - 1} \frac{1}{\nu!} f^{(\nu)}(\lambda_\nu)(T - \lambda_\mu I)^\nu \right] \frac{\prod\limits_{j=1}^{\mu-1} (T - \lambda_j I)^{K_j} \prod\limits_{j=\mu+1}^{M} (T - \lambda_j I)^{K_j}}{\prod\limits_{j=1}^{\mu-1} (\lambda_\mu - \lambda_j)^{K_j} \prod\limits_{j=\mu+1}^{M} (\lambda_\mu - \lambda_j)^{K_j}} \end{aligned}$$

(with obvious modifications in the last ratio when $\mu = 1$ or $\mu = M$). Here: i is the imaginary unit; $I = I_N$ is the identity operator on B; $(\lambda I - T)^{-1}$ is the resolvent operator of T at $\lambda \in \mathbf{C}\backslash\sigma(T)$, where $\{\lambda_1, \ldots, \lambda_M\} = \sigma(T)$; K_μ is the multiplicity of the eigenvalue $\lambda_\mu \in \sigma(T)$, $\mu = 1, \ldots, M$, so that $\sum_{\mu=1}^{M} K_\mu = N$; the Riemann contour integral in the above identity is the Cauchy-Riesz-Dunford operator integral for $f(T)$. While the first equality in the above identity is well-known to be true even in the case of infinite-dimensional Banach space B, the second equality in the above identity is new, to the best of the author's knowledge. The explicit form of its right-hand side, very remarkably, has the structure of an operator-valued Hermite interpolation polynomial. This new formula has a wide variety of applications, including, but not limited to, deriving a priori estimates for the operator resolvent of T and other analytic functions of T, of which special interest represents the exponent e^T of T. For $N = 1, 2, 3$ and 4 the characteristic equation of an $N \times N$ matrix T can be solved in radicals which allows new explicit representations and parametrizations of important matrix-based mathematical apparata of 4-dimensional physical space-time models. For example, the new formula allows for explicit parametrization of local and global diffeomorphisms and whole classes of differentiable manifolds diffeomorphically equivalent to an a priori given manifold. In the 3-dimensional case this has potential for applications, e.g., in solving the difficult problem of optimal registration of 3-dimensional images obtained by positron-emission tomography (PET) and used in brain surgery. The infinite-dimensional case is much more subtle and will be mentioned here only very briefly. In this case, the second equality in the above identity can only be obtained when the operator T is compact. In this case, to ensure the convergence of the infinite products which appear in the right-hand side, additional preconditioning is needed by a uniformly bounded strongly continuous operator (semi)group $e^{\varphi(T)}$, where φ is a polynomial of degree depending on the rate with which the eigenvalues of the compact operator T tend to zero.

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Oscillatory Properties of First Order Neutral Delay Impulsive Differential Equations with Constant Coefficients

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This paper is dealing with the oscillatory properties of the first order neutral delay impulsive differential equation of the form

$$\frac{d}{dt}[y(t) - cy(t-h)] + py(t-\sigma) = 0, \quad t \neq \tau_k, \quad k \in N$$
$$\Delta[y(\tau_k) - cy(\tau_k - h)] + p_k y(\tau_k - \sigma) = 0, \quad k \in N$$

where the deviations h and σ and the coefficients c, p and p_k are constants. The points $\tau_k \in (0, +\infty)$, $k \in N$ are fixed moments of impulsive effect (jump points), where the unknown function reveals its discontinuities of first kind as jumps. We manifest the jumps of the unknown function $y(t)$ in such points of impulsive effect by the notation

$$\Delta[y(\tau_k) - c_k y(\tau_k - h)] = \Delta y(\tau_k) - c_k \Delta y(\tau_k - h), \quad \Delta y(\tau_k) = y(\tau_k + 0) - y(\tau_k - 0).$$

Some criteria for oscillation of the solutions of such type of equations are obtained, allowing the coefficient c to be an arbitrary positive real number. It is considered, among other things, how the appearance of impulsive effects in an ordinary neutral differential equation can cause or destroy the oscillation of its solutions.

Examples with concrete equations and their particular solutions are given in order to demonstrate the obtained results.

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Histogram Optimal Multitresholding

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The paper concerns the optimal number of classes in one-dimensional (1D) recognition tasks that can be reached, e.g., in image segmentation by intensity. Effective methods for segmentation are known using image histogram thresholding, especially when the number M of the classes (intensity levels to group the image pixels) is preliminary given. Besides, by $M > 2$ and if the number of the whole picture intensities is relatively not very large (e.g., 256), the most of the methods allow effective extensions for histogram multi-thresholding, Otsu (1979, 1980), Kurita, Otsu, Abdelmalek (1992), Dimov, Gluhchev (1997). And, when M is unknown, programmers look for some heuristic evaluations of M. Generally, the paper problem solved can be defined as follows: to compute the minimal number K of Gaussian distributions $N(\mu_i, \sigma_i)$, $i = 1, 2, \ldots, K$, approximating a given 1D histogram in the frames of given admissible error.

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Cluster Analysis for Determination of the Influence of Basic Laser Parameters on Copper Bromide Vapor Laser Efficiency

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A copper bromide vapor laser of wavelengths 510.6 nm and 578.2 nm is the object of this research. The aim of the work is the investigation of relative influence of the basic laser parameters just like input power, geometric dimensions, hydrogen gas pressure etc. on laser efficiency. A big quantity of experimental data is used from the Metal Vapor Laser Laboratory of Georgi Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences. The data is processed by means of clustering methods. Ten basic laser parameters are grouped by different methods of similarity. Their influence on laser efficiency is determined. A comparison with other similar studies is done.

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Numerical Study of Linear Stability of Scalar-tensor Born-Infeld Black Holes

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Phases of scalar-tensor black holes coupled to non-linear electrodynamics where recently obtained and thermodynamical analysis of the obtained solutions was performed. Our task is to study numerically linear stability of the solutions. The obtained singular Sturm-Liouville problem is solved with a modifications of the shooting method. Results are obtained and they are in good agreement with the predictions of the thermodynamical analysis.

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Solvability for a Class of Systems of Second-Order Differential Equations by Using Hankel Transform

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In this paper we solved a linear system of ordinary differential equations with some initial and boundary conditions. Applying Hankel transform convert system of ordinary differential equation to a linear system of 2 by 2 without any derivation. After solving this system by using Hankel inverse transform we can find solutions of first system.

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Capacity Choice under Demand Uncertainty

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The sequential choice of capacity and quantity by firms in a strategic environment has been studied in the literature of Industrial Organization. This paper addresses the question of capacity choice under demand uncertainty in a mixed duopoly market consisting of one private firm and one public firm. We consider a two-stage game where firms choose capacity in the first stage without knowing the demand, and output in the second stage knowing the realized demand. We get that both symmetric and asymmetric outcomes can be realized.

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Endogenous Timing in a Differentiated Duopoly

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The model introduced by Stackelberg (1934) is one of the most widely used models in Industrial Organization for analyzing firms' behavior in a competitive environment. It studies the strategic situation where firms sequentially choose their output levels in a market. The question we ask is: Do first movers really have strategic advantage in practice? The belief of first-mover advantage was widely held among entrepreneurs and venture capitalists, but is now questioned by numerous practitioners. In this paper, we consider a quantity-setting duopoly model, and we study the decision to move first or second, by analyzing the implications of the market demand uncertainty, and also of the 'own' price effect.

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Particles in Creeping Flow Near a Slip Wall

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The no-slip boundary condition on a wall is the classical one for a viscous flow. But it has been recently recognized that a slip applies at small scales, e.g. on hydrophobic surfaces. Slip is characterized by a slip length, that is the distance at which a fictitious no-slip plane is moved away from the fluid to represent the slip plane in a shear flow. The slip length may be measured by following the motion of nanosized spheres close to such a surface. The motion of particles close to a slip surface also has applications in microfluidics, separation techniques in analytical chemistry, etc.

With these applications in view, this paper considers a spherical solid particle in creeping flow close to a slip wall. The flow along the wall is locally approximated by a pure shear flow and the sphere is translating and rotating. The various perturbed flow fields around the sphere are solved analytically using the bispherical coordinates technique. The hydrodynamic force and torque on the sphere are obtained in terms of the slip length. Results are provided with a precision better than 10^{-7} , even for a small gap down to 10^{-4} sphere radius.

The translational and rotational velocities of a freely moving sphere in a pure shear flow near a slip wall are then calculated. Interpolation formulae are provided. The hindered Brownian diffusion of a freely rotating sphere close to a slip wall is characterized by a diffusion tensor, the coefficients of which are derived.

Finally, the Aris-Taylor dispersion of Brownian particles in a shear flow near a slip wall is calculated from the advection-diffusion equation, using the expression for the particle velocity. For this purpose, the equation is first Fourier-transformed in the direction of vorticity. The transformed equation is solved by a finite elements package. Since it is two-dimensional, a refined mesh may be used.

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Stability in Liquid Systems with Interfaces

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Influence of gravity, Marangoni effect and Korteweg stresses on the stability of different viscous flows such as interaction between fluid particles, liquid binary mixture and rivulet flow is considered. The influence of the Marangoni effect on the equilibrium separation and migration velocities of the drops freely migrating under the action of gravity was studied. A critical radii ratio separating stable and unstable stationary configurations was found. The influence of gravity, solutocapillary effect and Korteweg stresses on the stability of a two-layer binary fluid system with diffuse interface is studied. Parameter regions of long-wave instabilities are found. The stability of the rivulet flowing down an inclined plane is investigated in the presence of cocurrent gas flow and variable gravity. The linear stability analysis of the system is carried out in the general case and the longwave limit of the general dispersion relation is derived.

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Theoretical and Numerical Analysis of the Fedorenko Finite Superelement Method

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Finite superelement method (FSEM) was suggested by R.Fedorenko in 1974. It claims to solve a wide range of "bad"problems, having sharp domain singularities or inhomogenities. Our last work presented the theoretical algorithm, which allows to construct and investigate FSEM approximations. A connection between FSEM and projectional methods was established, theoretical a priori error estimates were obtained, and the computational efficiency was confirmed in a range of solutions of physical problems.

This work concerns development and analysis of the FSEM referenced to the physical processes in conditions of strong media inhomogenities. A definition of FSEM is proposed, the general theory is briefly explained. The variants of method are considered. They are based on the increasing order of basis boundary polynomials, having an approximation property. Test results of these variants for the solution of 2-D and 3-D problems are given. Theoretical a priori error estimates are obtained by the example of the model problem. The problem of local regularity of numerical solution near corners are shown as well.

The work was done under partial financial support of Russian Fund of Basic Research (project 09-01-00151).

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An Immersed Interface FEM for Elliptic Problems with Local Own Sources

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A second order immersed interface finite element method for elliptic problems with local own sources is presented. The solution of the problem is continuous, but as a result of the local sources the jump of the flux depends on the solution (typically – nonlinear). The numerical method uses Cartesian grids and the standard linear basis functions are modified near the interface, so that the jump conditions are fulfilled. The case of discontinuous coefficients is also studied. Several test examples are presented to demonstrate the accuracy of the method.

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Charged Black Holes with Massive Scalar Field

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In the present work static and spherically symmetric magnetically charged black holes coupled to non-linear electrodynamics in the scalar-tensor theories of gravity with massive dilaton are investigated numerically. As a result of the numerical experiments some general properties of the solutions were found. The presence of potential for the scalar field leads to a different causal structure of the black hole solutions in comparison to the massless case.

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Spinor Bose-Einstein Condensates and Soliton Solutions of Multi-component NLS Models

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Consider BEC's of alcali atoms in the $F = 1$ hyperfine state, elongated in x direction and confined in the transverse directions y, z by purely optical means. The dynamics of this assembly of atoms is described by a 3-component normalized spinor wave vector $\mathbf{\Phi}(x,t) = (\Phi_1, \Phi_0, \Phi_{-1})^T(x,t)$ satisfying the nonlinear Schrödinger (MNLS) equation [1], [2]:

$$\begin{aligned} &i\partial_t\Phi_1 + \partial_x^2\Phi_1 + 2(|\Phi_1|^2 + 2|\Phi_0|^2)\Phi_1 + 2\Phi_{-1}^*\Phi_0^2 = 0, \\ &i\partial_t\Phi_0 + \partial_x^2\Phi_0 + 2(|\Phi_{-1}|^2 + |\Phi_0|^2 + |\Phi_1|^2)\Phi_0 + 2\Phi_0^*\Phi_1\Phi_{-1} = 0, \\ &i\partial_t\Phi_{-1} + \partial_x^2\Phi_{-1} + 2(|\Phi_{-1}|^2 + 2|\Phi_0|^2)\Phi_{-1} + 2\Phi_1^*\Phi_0^2 = 0. \end{aligned} \tag{1}$$

This model has natural Lie algebraic interpretation and is related to the symmetric spaces $\mathbf{BD.I} \simeq \mathrm{SO}(5)/\mathrm{SO}(3)\times\mathrm{SO}(2)$. It is integrable by means of inverse scattering transform method [3], [2]. Using a modification of the Zakharov-Shabat 'dressing method' we describe the different classes of soliton solutions of eq. (1), see [2] and the references therein.

In addition we present new families of stationary solutions for generalized equations of Bose-Fermi mixtures [4] with an elliptic function potential with modulus k. We also discuss particular cases when the quasiperiodic solutions become periodic ones. In the limit of sinusoidal potential ($k \to 0$) our solutions model a quasi-one dimensional generalized quantum degenerate Bose-Fermi mixture trapped in an optical lattice. The other limit $k \to 1$ provides solutions expressed by hyperbolic functions (vector solitons). Thus we obtain in a unified way quasi-periodic and periodic waves as welll as solitons.

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Classifying the Basic Parameters of Ultraviolet Copper Bromide Laser

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The performance of deep ultraviolet copper bromide lasers is of great import-ance because of their applications in high-precision processing of new materials, high-resolution laser lithography in microelectronics, high-density optical recording of information, laser-induced fluorescence in plasma and wide-gap semiconductors and, etc. In this paper we present a statistical study on the classification of 12 basic lasing parameters, by using different agglomerative methods of cluster analysis. The results are based on a big amount of experimental data for UV Cu^{+} Ne-CuBr laser with wavelengths 248.6 nm, 252.9 nm, 260.0 nm and 270.3 nm, obtained in Georgi Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences. The relevant influence of parameters on laser generation is also evaluated. The results are applicable in computer modeling and planning the experiments and further laser development with improved output characteristics.

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User-defined Service in g-Lite

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Service-oriented architecture (SOA) is an architectural model for developing reliable distributed systems, which functionality is provided as services. G-Lite is a middleware for grid computing that provide low and high level services for access to grid resources. Part of the services which, the environment exposes is service-enabled (follows the principles of SOA) and other part is not. This makes the environment partially service-oriented and creates difficulties with development of grid applications. From SOA principles' point of view, the problems in g-Lite are related with difficultness and lack of rules for creating, registering, deploying and certifying own grid service in the environment. In this article we discuss these issues and provide a simple solution to avoid these problems.

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Modeling of Cylindrical Couette Flow of Rarefied Gas. The Case of of Outer Rotating Cylinder

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The cylindrical Couette flow of a rarefied gas is studied in the case when the outer cylinder is rotating while the inner cylinder is at rest. Velocity, density and temperature profiles are investigated by a Direct Monte Carlo Simulation method and a numerical solution of the Navier-Stokes equations for compressible flow is found. The results obtained by both methods are: in an excellent agreement at a small Knudsen number Kn=0.02; in a satisfactory agreement at Kn=0.1 and they vastly differs each other at a moderate Kn=0.5. The comparison shows that the continuum approach can be used successfully for calculations of non-planar isothermal rarefied gas flows at small Knudsen numbers Kn<0.1. These results are important for applications in non-planar microfluidic problems.

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Numerical Algorithm for Constructing Jet Flows, of a Liquid of the Hydrodynamic

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This work dedicated learning simple jet flows in incompressible liquid.

It is learning planar potential flow incompressible liquid in simply connected domain, consist a polygon and jet flows is leaning endpoint of a polygon. Thus, $P(T)$ coincides with P if T satisfies the functional equation ?

$$g(T, \alpha) = l,\ g = (g_1, ..., d_n),\ g_k = \int_{t_k}^{t_{k+1}} |\Pi(t) M(t) dt,$$

where $l = (l_1, ..., l_n)$ is the vector of side length of P and $\alpha\pi = (\alpha_1 ... \alpha_n)\pi$ is the vector of its interior angles.

This proved local uniqueness of the solution equation describing these schemes.

The algorithm is constructed and is proved his convergence.

This work was partly supported by the Russian Foundation for Fundamental Research (Grant No. 09-01- 98001-r_Siberian _a) and “Mobility of vying scientists” (No. 09-08-90706-mob)

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Analytic Kerr-Sen Black Hole Gravitational Lensing in the Weak Deflection Limit

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We present approximate, analytical solutions of the equation of motion for a ray of light in the Kerr-Sen metric. The deflection angle is found and the gravitational lens equations are constructed. In order to investigate the gravitational lensing we compute the position and the magnifications of the individual images as well as the critical curves

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Numerical Investigation of Josephson Junction Structures

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Multilayered Josephson Junction Structures (JJS) form an interesting physical system where both nonlinearity and interactions between subsystems play an important role. Such systems allow to study physical effects that do not occur in single Josephson junction.

The Sakai-Bodin-Pedersen model – a system of perturbed sine-Gordon equat-ions – is used to study the dynamic states of the JJS. The corresponding static problem is numerically investigated as well. In order to study the stability of possible static distributions a Sturm-Liouville problem is generated and solved.

The dependence of the magnetic flux static distributions and their stability on the inductive coupling parameter are investigated. The transitions from static to dynamic state and the scenario of these transitions are analyzed depending on the same parameter and on the dissipation coefficient. Different physical characteristics – the potential and kinetic energies of the structure, the partial energies of the substructures and the full magnetic flux are calculated and interpreted.

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Numerical Solvers for Generalized Algebraic Riccati Equations

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The following constrained nonlinear algebraic equation (Definition 2.2 [1]) with the $m \times m$ unknown symmetric matrix X, is called a generalized algebraic Riccati equation (GARE). Here Z^+ denotes the pseudo-inverse of Z.

$$\begin{aligned}
&A^T X + XA + Q + C^T XC \\
&\qquad - (XB + C^T XD + L)(R + D^T XD)^+ (XB + C^T XD + K)^T = 0 \\
&[1 - (R + D^T XD)(R + D^T XD)^+](XB + C^T XD + L)^T = 0 \\
&(R + D^T XD) \geq 0.
\end{aligned} \tag{1}$$

Thus, following the classical linear quadratic theory we know that the GARE (1) can be solved via the following convex optimization problem:

$$\begin{aligned}
&\min(-\mathrm{Tr}(X)) \\
&\begin{pmatrix} A^T X + XA + C^T XC + Q & XB + C^T XD \\ (XB + C^T XD)^T & R + D^T XD) \end{pmatrix} \geq 0.
\end{aligned} \tag{2}$$

We consider and study several iterations for solving the equation (1) and optimizat-ion problem (2). The Newton method and the Lyapunov iteration applied to equation (1) and LMI approach to solve (2) are studied. We will compare the numerical effectiveness of the corresponding solvers. Numerical examples are used to demonstrate the performance.

This paper was financially supported under the St. Kliment Ohridski University of Sofia research project 2009.

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Use of the Two-Grid Method for a Heat Radiation Problem

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We consider the following nonlinear boundary value problem, which in particular describes a non-stationary radiative heat transfer

$$-\nabla.(k\nabla T) = f \quad \text{in} \quad \Omega,$$
$$\frac{\partial u}{\partial \nu} + H(x,y,u) = \int\limits_{\Omega} h[u(\xi)]\varphi(\xi,x,y)d\sigma(\xi) + g, \quad (x,y) \in \partial\Omega.$$

Here $\Omega = \bigcup\limits_{j=1}^{J} \Omega_j$, where Ω_j are bounded Lipschitz domains in $\mathbb{R}^2$ and $\overline{\Omega}_i \cap \overline{\Omega}_j = \emptyset$ for $i \neq j$. This paper is concerned with the solution of the nonlinear system of equations arising from FEM approximations of the problem. We employ Newton and Picard methods to develop a new version of the two-grid method originated from O. Axelson (*Appl. of Math.* **38**(4-5), 1993) and J. Xu (*SIAM Sci. Comp.* **15**(1), 1994).

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Numerical Analysis of Laplacian Differentialf Equations in Digital Image Processing Utilization for Efficient Environmental Impact f Assessment and Sustainable Development

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This paper analyzes the solution of Laplacian differential equation in several numerical schemes. Moreover, useful applications are made by the use of proper Laplacian modular forms, masks in digital image processing applications. The numerical analysis for the solution of Laplacian differential equations is focused on finite difference and finite element solutions so as to produce weak, strong and very strong image processing filters. Useful case studies and conclusions are made for the utilization of image processing results in geographic information systems, environmental impact assessments, sustainable development and public health protection.

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Application of the Fröbenius Method to the Schrödinger Equation

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The power series method is adapted to compute the spectrum of the radial Schrödinger equation $[-\frac{1}{2r}\frac{d^2}{dr^2}r + \frac{l(l+1)}{2r^2} + V(r)]u(r) = Eu(r)$. Considering two ways of imposing the boundary condition at $r = R$, namely $u(R) = 0$ or $\left.\frac{du(r)}{dr}\right|_{r=R} = 0$, we determine the lower and upper bounds for the eigenvalues as zeros of the calculable functions. This may be performed to arbitrary accuracy, since both functions are given by polynomials. For increasing R, the exact bound-state energy is approached from both sides monotonically. The generalization to the case of several coupled equations will be discussed and a few case studies will be presented for illustration.

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Single and Sub-cycle Optical Pulses in Air and Vacuum: Linear and Nonlinear Regime

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We present an analytical approach to the theory of optical pulses with superbroad spectrum propagated in isotropic media. In linear regime, new classes of exact 3D+ 1 unidirectional solution are presented. In nonlinear regime, for optical pulses with power, little above the critical for self-focusing, particle-like solutions of nonlinear amplitude equation and nonlinear wave equation are found.

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About Weak Convergence of Difference Schemes in Calculations of Unsteady Waves Through the Shocks

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Explicit two layer in time shock capturing difference schemes of high order are considered for hyperbolic systems of conservation laws. The approximation by this schemes of ϵ-Hugoniot conditions is researched (this conditions join values of exact solution at the boundaries of its shock ϵ-neighbourhood). The next theorem is proved: the order of the schemes approximation of the ϵ-Hugoniot conditions coincide with the order of its classical approximation if scheme has smooth functions of difference fluxes. Numerical experiments show that high order difference schemes which have smooth functions of difference fluxes (in contrast to its TVD modifications) retain high order of weak convergence through the unsteady shocks.

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Helmholtz Equation in Domains Bounded by Closed Curves and Open Arcs

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Boundary value problems for the Helmholtz equation are studied in planar domains bounded by closed curves and open arcs. Either Dirichlet or Neumann bondary condition is specified on the whole boundary (i.e., on both closed curves and open arcs). Theorems on existence and uniqueness of a classical solution are proved. The integral representation for a solution in the form of potentials is obtained. Each boundary value problem is reduced to the uniquely solvable Fredholm equation of the 2-nd kind and index zero for the density in potentials. Dirichlet and Neumann problems for the propagative Helmholtz equation are studied for exterior domain [5–8], while problems for dissipative Helmholtz equation [1–4] are studied in both interior and exterior domains. Problems in domains bounded by closed curves and problems in the exterior of open arcs in a plane are particular cases of our problems.

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Friction-induced Resonance of a Stochastic Oscillator

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Stochastically driven harmonic oscillators have been successfully applied to describe a wide variety of problems in nature. In this work influence of the friction coefficient on the long-time behavior of the first two moments and on the signal-to-noise ratio in the output signal of a harmonic oscillator with a fluctuating frequency subjected to an external periodic force and an additive thermal noise is considered. The colored fluctuations of the oscillator frequency are modeled as a three-level Markovian telegraph noise. This linear model of a noisy harmonic oscillator enables exact solutions for moments of the output signal and predicts some unexpected effects in the behavior of the stochastic resonance (SR) characteristics. The main purpose of this work is to demonstrate, based on exact expressions, that the resonance is manifested in the dependence of SR characteristics (such as spectral amplification and signal-to-noise ratio) upon the friction coefficient. The advantage of the latter effect is that the control parameter is the damping coefficient, which can easily be varied in possible experiments as well as potential technological applications, e.g. a variable resistor in electric oscillator devices. For this reason, we believe that the results of this work open up new possibilities to design experiments and observations in the field of SR.

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Multiresolution Scheme for Multiphase Flow

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In order to reduce the number of numerical flux computations in the multiphase equations, the multiresolution approach is proposed. The method is based on the analysis of the solution's regularity according to various levels of resolution. This technique provides a very useful tool for removing superfluous calculation where solution is over resolved. An accurate and efficient numerical method coupled with a multiresolution strategy, for predicting the evolution of flow phenomena, is presented. Numerical predictions of transient and steady flow problems in pipelines are compared to available experimental data.

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Nonlinear Dynamics of Axonemal/Flagellar Systems and Solitons

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Bio-membranes, nerves, filaments, flagella, cilia, etc. can be mathematical modeled by nonlinear (possible integrable) systems of differential equations, leading to the possibility of soliton or solitary waves propagation in such structures. In this talk we will present the general frame of a nonlinear bio-mathematical approach on the topic, and complete it by two examples. One application is provided by using the Bernoulli-Euler model for an axoneme whose dynamics is controlled by internally generated torques. We make predictions about its knoidal functions-like shapes, including possible solitary waves in the curvature. The shapes are related to the distribution of molecular motors activity, trying to explain certain swimming patterns of these cells. By using this model we also explain the experimentally observed constant behavior of the ratio between the length and the chord of bents in such axonemal systems. Another application relates to biological membranes that display (nonlinear) chain phase transformations close to temperatures of physiological interest allowing solitary waves propagation along electrically excited nerves. By solving analytically the dynamical equations describing the propagation of subsonic solitary waves along the membrane of a nerve we show the existence of shelve solitons, doublets and breathers. We investigate the possibility (experimentally proved) that two opposite solitary waves can annihilate each other at the center of the nerve.

This work is in collaboration with the Membrane Biophysics Group, Niels Bohr Institute, Copenhagen, and Laboratoire des Proteines, Université Paris 6.

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On the Structure of Biomathematical Models

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In recent times mathematical models as applied to biology have proved very useful in the understanding of biological systems. These models can be of discrete nature in which case they are described by difference equations, on the other hand they can be of a continuous nature in which case they are described by ordinary differential equations.The class of biological systems we investigate in this paper involve species interaction, that is interaction between different biological populations. These models can be used to investigate predator-prey interactions, or normal and abnormal cell interactions in epidemiology. We study in particular non-linear systems as well as the stability of the equilibria.

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Characteristic Line Scheme for Hierarchical Size-structured Model with Nonlinear Growth, Mortality and Reproduction Rate

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In the present work we propose a new scheme based on characteristic lines for solving hierarchically structured population model with nonlinear growth, mortality and reproduction rate. The scheme is stable and shows second order of approximation in x and t. The idea of the method, proposed from Marinov, Sakai, and Marinova for solving of the advection equation in a conservative form, is not to follow the characteristics from the initial condition, but for each time-step to recover trough the characteristics the position at the previous time level of the functional values that arrive at a grid point on the net tile level. Numerical results show second order of convergence of the new schemes. The scheme is validated for two exact solution: a continuous and a discontinuous. In addition, we compare the results of the new scheme with two known numerical numerical schemes for the model under consideration.

The new scheme is very efficient for investigating the propagation of disturbances (waves) in size-structured model with nonlinear growth, mortality and reproduction rate. We have been able to get accurate prediction for the propagation of a disturbance from the boundary inside the region for the case of strongly nonlinear coefficients.

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Cubic Spline Approximation of the Coefficient f in Euler-Bernoulli Equation from Over-posed Data

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In this work we consider the simplest form of Euler-Bernoulli equation

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(\sigma(x)\frac{\mathrm{d}^2 u}{\mathrm{d}x^2}\right) = f(x)$$

in a rectangular domain. If the coefficient $\sigma(x) > 0$ and the function $f(x) \geq 0$ are given, under proper initial and/or boundary conditions, the problem possesses a unique solution, usually referred as a direct solution. In practice, there exist lots of interesting problems, in which the coefficient $\sigma(x)$ is not exactly known. A new, so called inverse problem, appear to find simultaneously the solution u and the coefficient σ. The method we use to treat the inverse problem is proposed by C. I. Christov and called Method of Variational Imbedding (MVI). Recently, the MVI was applied to the problem for identifying the coefficient σ in the case when it is piece-wise constant and piece-wise linear function. Now we consider the case when the coefficient is a piece-wise cubic function. The numerical results confirm that the solution of the imbedding problem coincides with the direct simulation of the original problem within the second order of approximation.

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Rapid Convergence via Generalized Quasilinearization Method for Impulsive Differential Equations

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In this paper we have extended the method of generalized quasilinearization to nonlinear impulsive differential equations with initial conditions. We consider the case when the second derivative of the forcing function $f(t, x(t))$ and the function of impulsive effect $I_k(x)$ with respect to x exist such that they are one sided Lipschitzian in x and are monotone in x. We have considered natural lower and upper solutions under suitable conditions and obtained two sequences which converge uniformly and monotonically to the unique solution of the nonlinear impulsive differential equations. We further prove that the rate of convergence is cubic. Next we have obtained a fourth order convergence for nonlinear impulsive differential equations when the nonlinearities satisfy a regularity, a monotonicity, and a Lipschitz condition. Furthermore, one can extend these results to obtain nth order convergence (n being an even or odd number) for nonlinear impulsive equations with initial conditions.

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Time-resolved Optogalvanic Signal Modeling in Ne Hollow Cathode Plasma

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The dynamic optogalvanic effect represents a change of the plasma conductivity due to absorbtion of short (ns) pulses of resonant light. This perturbs the ionization balance in the plasma by redistribution of the populations of the energetic levels involved in the illuminated optical transition. The time shape of the optogalvanic signals is able to give quantitative information for the rates of different collision processes in the plasma, responsible for the dynamics of optogalvanic effect format-ion. In the present work, a mathematical rate equation model is presented including the various processes contributing to the generation of optogalvanic signals in Ne hollow cathode discharge. It has been used to analyse the experimentally registered time-resolved optogalvanic signals of Ne $2p_{10}$–$4d_3$ (533.08nm) and $1s_4$–$2p_1$ (540.05nm) atomic transitions at different discharge current values. The decay rates of the levels contributing to signal formation have been determined using non-linear fit of the experimentally measured dynamic optogalvanic signals with theoretically obtained function.

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I-Binomial Risk Model

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The Inflated-parameter binomial process (I-Binomial) as a compound binomial process is defined. Some basic properties are given. We consider the risk model in which the counting process is the I-Binomial process. It is called I-Binomial risk model. The joint probability distribution of the time to ruin and the deficit after ruin occurs is studied. The particular case of exponentially distributed claims is given.

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Mathematics for Nuclear Waste Management

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Electricity production, in France, comes mostly from nuclear energy. Proper solutions for radioactive waste repository must be found: mathematical models describe the radioactivity transfer from repository to living species and environment in general. Probabilistic methods can be used to compute safety thresholds related to the radioactivity transfer in order to decide what kind of repository must be used for each radioactive waste type.

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Integrable Many-body Systems and Bianchi Cosmological Models

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It is shown that the geodesic motion on the $GL(n;R)$ group manifold endowed with a bi-invariant metric corresponds to a generalizations of the n-particle Calogero-Moser-Sutherland model and the non-periodic Toda chain. Using the methods of Hamiltonian reduction these many-body systems are related to Bianchi cosmological models.

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Sequences of Dipole Black Rings and Kaluza-Klein Bubbles

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We discuss a new class of exact static solutions to the Einstein - Maxwell equation in five dimensions with Kaluza-Klein ($M^4 \times S^1$) asympotics. The solutions describe sequences of Kaluza-Klein bubbles and dipole black rings and are obtained by 2-soliton transformations from a seed solution describing neutral sequences of Kaluza-Klein bubbles and black holes. In contrast to the seed solution, however, the sequences discussed by the authors are charged, thus being more general. The consequences of the presence of charge are explored obtaining the Smarr-like relations and the first law of black hole thermodynamics.

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Optimized Spectral Approach to Quantum-Mechanical Problems

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Spectral approach consists in representing a trial solution of the differential equation as a linear combinations of the basis functions. When applied to solving quantum-mechanical Schrödinger problems, this procedure bears the name of the Rayleigh-Ritz or Configuration Interaction method. We review research efforts aimed at ensuring optimal convergence of the method by an appropriate choice of the expansion basis. In particular, introducing nonlinear parameters into the functions of the basis and various strategies of adjusting their values will be addressed. We formulate the optimized Rayleigh-Ritz method, where the values of the parameters are fixed so as to minimize the trace of the variational matrix. This universal method proves very economical and enables determination of bound-state energies of a particle trapped in various potentials to arbitrary accuracy. In the few-particle case, the method leads to accurate determination not only of bound-states, but also of natural orbitals for trapped Fermi and Bose systems with different interparticle interactions.

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Singularities at a Solid-fluid-fluid Triple Contact Line, for an Elastic Solid

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At a solid-fluid-fluid triple contact line (e. g., solid-liquid-gas), the action of the fluid-fluid surface tension on the solid is a force concentrated on that line, which produces singularities of the elastic strain and stress tensor fields in the solid. Equilibrium thermodynamics, including surface effects (definition of the "surface stress" tensor), leads to a variational problem with mobile discontinuities (at the triple line). The resulting singularities can be illustrated in the case of a solid thin plate, treated in the frame of the nonlinear von Karman model. The equilibrium equations involve discontinuity jumps (at the triple line) of the derivatives $\mathrm{D}u$, D^2v and D^3v, where u and v respectively denote the in-plane and the normal displacements of the plate. In addition, the classical capillary Young's equation (which gives the angle of contact, in the fluid) is strongly modified.

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Finite-Difference Schemes for Hyperbolic Systems of Conservation Lows

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Presently explicit two layer in time TVD and ENO schemes are widely used for shock-capturing computations of discontinuous solutions of hyperbolic systems of conservation laws. In this report it is shown that in the region of influence of nonstationary shock all these schemes have the order of convergence not higher then the first one, independently of its formal accuracy in the sense of Taylor series expansion on smooth solutions.

The theory of construction of shock-capturing schemes, reserving the high order of convergence in the region of shock influence, is proposed. It follows from the theory, that really shock-capturing difference scheme must have high order of weak approximation on its difference solutions. A new high order scheme whish has a third order of weak approximation is constructed. The results of test calculations are presented in order to demonstrate the advantage of this new scheme in comparison with TVD schemes.

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Two-Gradient Convection in a Vertical Slot with Maxwell-Cattaneo Heat Conduction Law

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We study the effect of the non-locality of the Maxwell-Cattanneo (MC) (see, the formulation for moving media in [1]) law of heat conduction on the 1-D flow in a vertical slot subject to both vertical and horizontal temperature gradients. The gravity force is allowed to oscillate which gives the opportunity to investigate the quantitative contribution of the heat inertia as epitomized by MC law. We use a spectral expansion with Rayleigh's beam functions as the basis set, which is especially suited to this kind of problems.

The first part of the investigation is concerned with the eigenvalues and eigenmodes of the system. The addition of the time derivative via MC law increases the order of the system. We show that the MC part has a dissipative nature introducing a family of purely real negative eigenvalues. Yet it also also lowers the imaginary parts of the complex eigenvalues. Thus, the system has somewhat more oscillatory behavior when converging to the steady-state solution.

The fact that the frequencies of the eigenmodes are lower in the MC case manifest itself more conspicuously in the oscillatory regimes when an interplay between the forced frequency of the gravity modulations and the natural frequencies takes place. This interplay changes qualitatively the parametric instability diagram for low frequencies compared to the pure Fourier case investigated in [2], [3]. The expected pattern of alternating isochronous and subharmonic (period-doubling) bifurcation is observed for MC as for Fourier law, but the Fourier-law solutions are stable for larger values of the modulation amplitude than their M.C. counterparts.

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Stochastic Stability of the Viscoelastic Rotating Shaft

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The dynamic stability problem of a viscoelastic Voigt-Kelvin rotating shaft subjected to action of axial forces at the ends is studied. The shaft is of circular cross-section, it rotates at a constant rate about its longitudinal axis of symmetry. The effect of rotatory inertia of the shaft cross-section is included in the present formulation. Each force consists of a constant part and a time-dependent stochastic function. Closed form analytical solutions are obtained for simply supported bound-ary conditions. By using the direct Liapunov method almost sure asymptotic stability conditions are obtained as the function of stochastic process variance, retardation time, angular velocity, and geometric and physical parameters of the shaft. Numerical calculations are performed for the Gaussian process with a zero mean and variance s^2 as well as for harmonic process with amplitude H.

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Increasing the Calculation Speed of an Acceleration Scheme for an Age-Structured Diffusion Model

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In this paper we propose an optimized algorithm, which is faster compared to previously described finite difference acceleration scheme, namely the Modified Super-Time-Stepping (Modified STS) scheme for age-structured populat-ion models with diffusion. Keeping the accuracy of the Modified STS algorithm, we reduce its computational time almost two times, obtaining an additional speed-up. This makes the optimized method highly preferable for nonlinear and higher-dimensional problems.

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Oscillation Properties of Boussinesq Equation and Comparison with Other Fourth Order Equations

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In this paper we establish sufficient conditions for the absence of eventually positive solutions of Boussinesq equation:

$$z^{iv}(t) + \beta z''(t) + z^2(t) + \lambda z(t) = F(t),$$

where β and λ are proper constants as well as $F(t) \in C([T,\infty);\mathbf{R})$, $T \geq 0$ is a large enough constant. Further, we find sufficient conditions for oscillation of the equations:

$$z^{iv}(t) + \beta z''(t) + z(t)|z(t)| + \lambda z(t) = F(t)$$

and

$$z^{iv}(t) + \beta z''(t) + z^{2n+1}(t) + \lambda z(t) = F(t), \qquad n \in \mathbf{N}.$$

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Hybridization of the Vector Finite Element Method with the Boundary Integral Method for the Solution of Finite Arrays of Cavity-Backed Slot Antennas

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The problem under consideration is the accurate solution of a cavity-backed slot antenna that is flush-mounted on an infinite ground plane on top of which may reside a number of layered dielectric substrates. The cavity is excited through the use of a coaxial cable, microstrip line, waveguide, or just a simple delta gap. The governing equations for this type of radiation problem are the Maxwell equations which are discretized using a Galerkin type of approach in the context of the vector finite element method. The basis functions are such that they guarantee continuity of the unknown field across dielectric interfaces and satisfy the divergence condition in a source-free region. The vector finite element method is used to discretized the two curl time-harmonic Maxwell equations in the cavity domain, whereas the equivalence principle is used to introduce magnetic currents in the aperture. Therefore, the problem is split into an interior domain and an exterior domain; the latter is solved using a boundary integral approach through the use of the appropriate Green's function. An array of such antennas requires the use of an enormous amount of storage and computational time in order to solve the required matrix system. The number of unknowns may reach the order of tens of millions. One possible solution is to use periodic boundary conditions and treat the problem as an infinite array instead of a finite one, thus compromising accuracy even though this type of approach is highly efficient. The other approach is to use techniques that avoid storage of information related to repetitive geometrical features, and by using stationary/non-stationary iterative techniques one can solve the governing block matrix system quickly and quite accurately. Obtained results using stationary techniques such as the block Successive Under-Relaxation technique are quite promising and indicate a computationally efficient approach to solve such type of boundary-value problems.

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Two-zone Approach for Modeling a Steady Glass Fiber Drawing Process

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Two numerical methods are developed for modeling the steady drawing process of a glass fiber. Both methods are based on the one-dimensional version of the equations of motion coupled by the heat transfer equation. In addition to the conventional models the effects of axial heat conduction, radiative heat transfer and air-drag are taken into account. In the first method the drawing zone starts at the nozzle and continues down to the take-up wheel. In the second method we separate the drawing zone into two parts: melted (liquid) and solidified zones. These zones are then smoothly matched in an unknown section whose coordinate is treated as spectral parameter. This parameter is calculated using the continuous analog of Newton method combined by spline-collocation scheme. The results thus obtained are compared when calculating fiber radius, axial velocity and temperature. In addition the effects of Stanton and radiative numbers as well as the effect of air-drag are illustrated.

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On Fractional q-Integrals, q-Derivatives, Corresponding Equations and Applications

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The fractional differential equations (FDE), as generalizations of integer-order ones, are used in describing various phenomena in the science, especially in physics, chemistry and material science, because of their ability to describe memory effects. Two basic types of fractional derivatives, Riemann- Liouville and Caputo, produced two types of FDE.

Many of continuous scientific problems have their discrete versions. A way of the treatment is from the point of view of q-calculus. W.A. Al-Salam and R.P Agarwal introduced several types of fractional q-integral operators and fractional q-derivatives, always with the lower limit of integration equal 0. However, in some considerations, such as solving of q-differential equations of fractional order with initial values in nonzero point, it is of interest to allow that the lower limit of integration is variable. Our purpose in this paper is to generalize this theory in that direction.

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Numerical Simulation and Computational Analysis of Nonlinear Miscible Displacement Flows

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The formation and evolution of two-dimensional ($2D$) fingering patterns are frequently characterized at successive times by analyzing simulated one-dimensional ($1D$) transversely averaged concentration profiles. Such profiles are studied here for structural comparison between miscible density fingering (DF) and viscous fingering (VF). From numerical simulations it is known that VF and DF can evolve in a quite similar way, however, as each dynamics follow different evolution equations, some structural differences between them are expected, specially in the nonlinear regime. Although many physical properties influence the fingering pattern formation in VF and DF miscible displacements, there are few measurements on the averaged profiles that help to characterize their expected fine differences. Using the Gradient Pattern Analysis (GPA), a technique highlighting bilateral asymmetries in the growth of fingers, we introduce a complementary approach to study DF and VF structures. The typical range of gradient asymmetry coefficients (G_A) obtained for transverse concentration averaged profiles shows that GPA is a useful tool to characterize fine structural differences between viscous and density fingers. In order to validate G_A as a new measurement for fingering analysis, measures as mixing length and statistical skewness are also computed.

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Toward Absolutely Stable Finite Difference Schemes with the Positivity Condition

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So far we have been developing absolutely stable numerical schemes with positive difference coefficients for advection-diffusion equations. At first we have developed FVDM(Finite Variable Difference Method) for steady advection-diffusion equations, in which locations to evaluate numerical fluxes are optimized so that characteristics roots of the resulting finite difference equation may be positive. Then we have developed stable schemes ANO and COLE based on theoretical solutions for unsteady linear and nonlinear advection-diffusion equations, and the third-order polynomial scheme TOP optimized regarding numerical stability and truncation errors in the non-conservative form. Further we present a scheme CROSS to account for the cross-derivatives between x- and y-coordinates in two-dimensional flow fields, and a third-order scheme FLUX to calculate numerical fluxes in the conservative form so that the difference coefficients of resulting finite difference equation may be positive.

Numerical experiments have been performed, resulting in non-oscillatory solut-ions even for shock formation problems.

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Determining a Modern Energy Density Functional for Nuclei Using The Simulating Annealing Method

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The development of a nuclear energy density functional (EDF) for accurate predictions of properties of nuclei is very important for the study of properties of rare nuclei with unusual neutron-to-proton ratios that are difficult to produce experimentally and likely to exhibit interesting new phenomena associated with isospin, clusterization and the continuum. We have implemented the simulating annealing method (SAM) to the problem of searching for the global minimum in the hyper-surface of the chi-squared function which depends on the parameters of the EDF, developed using the Skyrme effective interaction. The parameters were obtained by a fit of an extensive set of experimental data to results of mean-field calculations. We constrained the values of the parameters by imposing conditions on nuclear matter properties.

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Computational Electronics on GRID: A Mixed Mode Carrier Transport Model

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The nano-era of the semiconductor electronics raises the necessity of simulation methods which describe the electron transport in ultra-small devices as a mixed mode quantum process. The latter accounts for both, quantum-coherent processes of interaction with the device potential, and phase-breaking processes of de-coherence due to scattering with phonons and other crystal lattice imperfections. The Wigner formulation of quantum mechanics is particularly convenient for mixed mode transport description, since it utilizes a phase space, where many classical notions are retained. In this picture the scattering can be accounted for in a straightforward way by using the Boltzmann collision models. While these models can be applied by using the well-developed classical simulation algorithms, the coherent counterpart gives rise to a heavy numerical burden. On contrary the alternative formulation of quantum transport in terms of Green's functions, is numerically efficient in the cases of coherent transport and problematic if phase-breaking processes are considered.

We propose and approach which combines the advantages of the two pictures: Green's function calculations of the of coherent transport determined by the boundary conditions in the semiconductor device provide the coherent Wigner function f_w^c. It is used into an equation for the correction Δf_w to f_w^c, obtained by a subtraction of the coherent Wigner equation from the general coherent/de-coherent counterpart.

Furthermore this equation is approximated by its classical limit. An evolution problem is obtained, where the initial condition is determined by f_w^c, while device boundaries become an ideal sink of carriers. For very small devices, where the carrier dwelling time (the time needed for a carrier to abandon the device through the boundaries) is accordingly small, already the initial condition may be considered as a sufficient correction accounting for the de-coherence effects.

We present a particle model based on the numerical Monte Carlo (MC) theory which has been derived for evaluation of the initial condition. The initial conditions involve multidimensional integrals. The MC algorithm is developed to estimates this integrals which require a lot of computational times. We propose efficient implementation of the algorithm on Grid environment in order to reduce computational effort. Some numerical results are presented.

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Optimal Control Problem for Some Non-linear Systems with Time Lag

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Time delays are frequently encountered in the behavior of many physical processes and very often are the main cause for poor performance and instability of control systems. The significant growth of interest in such systems is due to their various applications in control theory and automatic regulation , biology and ecology, biomedicine and others. In view of this, time delay systems is a topic of great practical importance which attracted a great deal of interest for several decades.

The purpose of this paper is to present an approximate method for the optimal control synthesis for systems of the form:

$$\begin{aligned}&\dot{x}(t) = A(t)x(t) + B(t)u(t) = \varepsilon f(t, x(t-h)),\\&0 \leq t \leq T, \quad x(0) = a_0, \quad x(\theta) = \phi(\theta), \quad \theta \in [-h, 0),\end{aligned} \tag{1}$$

where x is a coordinate vector from the Euclidean space $\mathbf{R}^n$, u is a control vector from $\mathbf{R}^m$, and matrices A, B have continuous and bounded elements. The small parameter $\varepsilon \geq 0$, constant time lag $h > 0$, vector a_0 and final time T are given. The function f is continuous with respect to all its variables, continuously differentiable with respect to its second argument, and for some constants $C_1, C_2 \geq 0$, satisfies the condition

$$|f(t, x(t-h))| \leq C_1 + C_2|x(t-h)|^2, \tag{2}$$

where $|\cdot|$ is the Euclidean norm. The continuous function ϕ is known.

Problem 1. *Find the control vector u which translates the initial system from the initial state a_0 to the final state a1 such that the criterion*

$$J(u) = \int_0^T |u(s)|^2 ds \tag{3}$$

is minimized.

This means that $x(0, u) = a_0$ and $x(T, u) = a_1$. We introduce the notation

$$z(t, s) = \exp \int_s^t A(\tau) d\tau, \quad G_1(t) = \int_0^t z(t, s)B(s)B'(s)z'(t, s) ds G^{-1} \tag{4}$$

where the prime denotes transpose.

It is known, that if G is a nonsingular matrix, then with $\epsilon = 0$, Problem 1 has an explicit analytical solution. We also assume that G is nonsingular. In the present work we present a new approach to Problem 1 with $\epsilon \neq 0$ and $h \neq 0$ under the above assumptions concerning the parameters of the system (1) to construct sequences of controls and trajectories in such a way as to approximate the optimal ones, and to obtain an error estimate and an upper bound of the parameter ϵ for which the proposed method is correct.

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Solitons in the Scalar-tensor Theories of Gravity with Non-linear Electrodynamics

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Compact object are a major field of interest in gravitational physics since they are a natural arena to study gravity in the strong field regime. Especially interesting is the possibility of existence of regular sources (as an alternative to the black holes and naked singularities) of strong gravitational field, i.e., soliton-like solutions. Such solutions have been obtained in General Relativity (GR) with non-linear matter models. Our aim is to obtain regular, soliton solutions in the scalar-tensor theories of gravity which are one of the most natural alternatives of GR. For the non-linear matter model we have considered non-linear electrodynamics.

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Fast Time Scale of Volatility in Interest Rate Models

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We are interested in term structure models for pricing zero coupon bonds under rapidly oscillating stochastic volatility. We analyze solutions to the generalized Cox-Ingersoll-Ross two factors model describing clustering of interest rate volatilities. The main goal is to derive an asymptotic expansion of the bond price with respect to a singular parameter representing the fast scale for the stochastic volatility process. We derive the second order asymptotic expansion of a bond price. The first two terms are independent of the variable representing stochastic volatility. They depend only on mean value and dispersion of the limiting distribution of the stochastic dispersion and other model parameters. The next term in the expansion depends also on the realized value of volatility and its averaged value depends nontrivially on the volatility distribution.

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Detection of Quasi-stationary Features in MHD Turbulence of Molecular Clouds Use the Time-scale Spectra

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Using two-dimensional Fourier transform the results of model calculations of space plasma temporal dynamics within its spatial structuring are obtained. It is shown that occurrence of molecular (plasma) clouds can be divided into two time phases. The first is connected with details of small angular sizes and theirs turbulence formation, fast increase of maximal size and power emission at short time interval. The second phase is characterized smooth, linear increasing of the size details, falling power emission oscillating, transition from fast chaotic movements to slow displacement and formation big ordered spatial structures. Disappearance of details with small angular sizes is observed.

The work was supported by the Royal Society UK-Russian International Joint Project, the grants RFBR 08-02-13633-ofi-c, 08-02-91860-KO-a and 08-02-92204-GFEN-a.

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Freezing of a Suspended Supercooled Droplet with a Heat Transfer Mixed Condition on Its Outer Surface

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The problem of water droplets freezing before or after their impact on solid surfaces is of major importance when modeling the aircrafts icing in wind tunnels. The object in this study is to estimate the freezing time of a suspended supercooled droplet in an air flow. It is known that freezing occurs into two steps: a short stage of rapid return to thermodynamic equilibrium, when the droplet becomes a water-ice mixture and a longer stage of its complete freezing. Mathematically, the second freezing step can be modeled by the one-phase Stefan problem. A convective heat transfer with ambient air is modeled here by a mixed boundary condition on the droplet outer surface. Assuming a spherical droplet, an asymptotic solution is developed for small Stefan numbers, while for arbitrary Stefan numbers a numerical solution based on the enthalpy method is constructed. The asymptotic and numerical solutions are compared to other authors experimental results.

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A Mathematical Model of Erythropoiesis Subject to Malaria Infection

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The red blood cell population is suppressed by malaria infection in several ways. This model accounts for the two that are most significant. First, the population is preyed upon by the parasites. Second, the production of a toxin called hemozoin, produced by the parasites, suppresses the development of erythroid precursors. A structured mathematical model of this process, in the form of two hyberbolic partial differential equations and four ordinary differential equations, will be presented. The finite difference scheme developed for the numerical solution of the model will also be presented. An outline of the proof of existence-uniqueness of solutions, via the finite difference scheme, will be given. Finally, some simulations of the model will be shown and their implications concerning treatment strategies will be discussed.

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Asymptotic Analysis in Dynamical Heat Transfer Problems in Heterogeneous Periodic Media

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The effective behavior of the solution of a nonlinear dynamical boundary-value problem modeling thermal diffusion in some heterogeneous periodic media is analyzed. We deal, at the microscale, with an ε-periodic structure, consisting of two parts: a fluid phase and a solid skeleton (reactive obstacles). In such a domain, we consider a heat equation, with nonlinear sources and with a dynamical condition imposed on the heterogeneous boundaries of the reactive obstacles. We are interested in describing the asymptotic behavior, as the small parameter ε, which characterizes the size of the reactive obstacles, tends to zero, of the temperature field in this periodic structure. Using an homogenization procedure, we prove that the effective behavior of the solution of our problem is governed by a new parabolic equation, with extra-terms coming from the influence of the nonhomogeneous dynamical boundary condition.

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Collision Dynamics of Initially Polarized Solitons in CNSE

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For the Coupled Nonlinear Schrödinger Equations (CNLSE)

$$\mathrm{i}\,\psi_t = \beta\psi_{xx} + \left[\alpha_1|\psi|^2 + (\alpha_1 + 2\alpha_2)|\phi|^2\right]\psi,$$

$$\mathrm{i}\,\phi_t = \beta\phi_{xx} + \left[\alpha_1|\phi|^2 + (\alpha_1 + 2\alpha_2)|\psi|^2\right]\phi,$$

we construct a conservative fully implicit scheme (in the vein of the scheme with internal iterations proposed in [1]) and implement our computations in complex arithmetic. The scheme conserves the "mass", momentum, and energy of CNSLE within the round-off error.

Collisions of solitary waves (quasi-particles) are investigated for different polarization angles and phases of the two solitons in the initial configuration. General initial elliptic polarizations (not *sech*-shape) include as particular cases the circular and linear polarizations. We elucidate numerically the role of nonlinear coupling on the quasi-particle (QP) dynamics. For various rates of nonlinear coupling parameter α_2 (so-called *cross-modulation*), we find that after the collision the carrier frequencies of the QPs change as a rule, i.e., the polarization angles of the emerging QPs are different from the original. We call such a phenomenon 'polarization shock'. We have uncovered many other different scenarios of the QP behavior upon collision. In some cases with linear initial polarization, the interaction actually causes a new QPs to appear, e.g., a standing soliton. Alternatively, in cases with elliptic initial polarization the dissolution of one of the QPs is observed due to energy trapping by the other. An interesting new result is that Manakov solitons with 45° initial polarization can emerge unchanged from the collision even for $\alpha_2 \neq 0$, provided that the initial phases of both QPs are equal to zero.

These effects enrich the knowledge about the intimate mechanisms of interaction of polarized two-soliton QPs in CNSE.

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Riesz Fractional Derivatives for Korteweg-de Vries Equation and Its Soliton Solutions

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Riesz fractional derivatives are well known for their role in studying existence and uniqueness for nonlinear partial differential equations and for the Korteweg-de Vries equation (KdV henceforth) in particular. New fractional properties are established for the fundamental solution of Cauchy problem for the linearized KdV. This study is based on the fractional derivative properties of the Airy functions. A soliton solution of KdV is chosen as an important example. Special function representations are obtained for Riesz fractional derivatives of this soliton and their Hilbert transforms. This family of functions turns out to be a fundamental system of solutions for a second order ordinary differential equation in self-adjoint form. Various analytic properties are established and graphic illustrations are given.

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Fast Iteration Algorithms for Solution of Exponential Schemes of Semi-Linear Singularly Perturbed 2D Reaction-Convection Equations

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In this paper we solve by exponentially difference schemes singularly perturbed nonlinear two-dimensional convection-diffusion equations in a channel domain. To find the solution of the nonlinear algebraic systems we investigate Newton and Picard methods. We propose a new version of the two-grid method solving in the first step, the nonlinear differential problem on a "coarse" grid of size H. In the second step, the problem is linearized around an appropriate interpolant of the computed solution on the first step and the linear problem then is solved on a fine grid of size $h \ll H$. It is shown that the algorithms achieve optimal accuracy as long as the mesh size satisfy $h = O(H^{2m}),\ m = 1, 2, \ldots,$ where m is the number of the Newton (Picard) iteration for the differential problem. The convergence of the discrete solutions is always ε – uniformly. We count the number of the arithmetical operations to illustrate the computational cost of the algorithms. Numerical experiments are discussed.

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Numerical Stability Analysis of the Discretized Advection-Diffusion Equation on Non-uniform Mesh Size Grids

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Although non-uniform mesh size grids are often used for actual engineering applications, numerical stability analysis of the finite difference equations on non-uniform mesh size grids has not been performed so far. Therefore numerical stability analysis of discretized advection-diffusion equations were performed for non-uniform mesh size grids by means of an eigenvalue analysis method. The stability domains in a typical non-uniform mesh size grids are provided quantitatively, and the domains are narrower as compared with those in case of uniform mesh size girds. Through the numerical stability analysis using the FTCS scheme, it is shown that the allowable maximum Courant number is less than the mesh size ratio to the averaged mesh size. Further, the stability domains are verified through numerical experiments for the advection-diffusion equations.

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Solitons and Concentric Rotating Black Rings with Dipole Charge

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We discuss soliton solution generating techniques in 5D Einstein-Maxwell gravity. Exact solutions describing two concentric rotating dipole black rings are constructed. The properties of the solution are investigated and some interesting physical effects are considered.

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Field Theoretical Approach to Dynamics of Deformation and Fracture

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Based on the field theoretical approach developed by physical mesomechanics (a recent gauge theory), an attempt is made to describe all the stages of deformation of solid-state media on the same theoretical foundation. The equation of motion governing mesoscopic elements in plastically deforming media has been derived from the mesomechanical field equation, and has been interpreted as representing the constitutive equation of plasticity. Recent analysis indicates that in the plastic regime, media exert two types of force in response to external loads; restoring force and energy dissipative force. The former is associated with shear resistance and causes the displacement field oscillatory. The latter is associated with strain concentration and causes the displacement field to be decaying. Fracture can be defined as the final stage of deformation where the medium loses the restoring capability. Experiments have been conducted to verify these formulations.

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DSP Algorithms for Fission Fragment and Prompt Fission Neutron Spectroscopy

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Digital signal processing (DSP) algorithms for fission fragment (FF) and prompt fission neutron (PFN) spectroscopy is described in present work. The gridded twin ionization chamber (GTIC) is used to measure the kinetic energy-, mass- and angular distributions of the FF in the ^{252}Cf(SF) reaction. Along with the neutron time-of-flight (TOF) detector the correlation between neutron emission and FF mass, energy is investigated. The TOF is measured between common cathode of the GTIC and the neutron detector (ND) pulses. Waveform digitizers (WFD) having 12 bit amplitude resolution and 100 MHz sampling frequency are used for the detector pulse sampling. DSP algorithms are developed as recursive procedures performing the signal processing, similar to those available in various nuclear electronics modules such as constant fraction discriminator (CFD), pulse shape discriminator (PSD), peak-sensitive analogue-to-digital converter (pADC), pulse shaping amplifier (PSA). To measure the angle between FF and the cathode plane normal of the GTIC a new algorithm is developed, having advantage over the traditional analogue pulse processing schemes. Algorithms for the pulse height measurement were described in previous reports, therefore in the present work they will be considered briefly. Algorithms are tested using both numerical simulation and the comparison of the data analysis of the ^{252}Cf(SF) reaction with the data available from literature.

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